

Mathematics

## Unit Statistics 1B

## Statistics

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## For this paper you must have:

- the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.


## Time allowed

- 1 hour 30 minutes


## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The final answer to questions requiring the use of tables or calculators should normally be given to three significant figures.


## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75 .
- Unit Statistics 1B has a written paper only.


## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A production line in a rolling mill produces lengths of steel.
A random sample of 20 lengths of steel from the production line was selected. The minimum width, $x$ centimetres, and the minimum thickness, $y$ millimetres, of each selected length was recorded.

The following summarised information was then calculated from these records.

$$
S_{x x}=2.030 \quad S_{y y}=1.498 \quad S_{x y}=-0.410
$$

(a) Calculate the value of the product moment correlation coefficient between $x$ and $y$.
(b) Interpret your value in the context of the question.

2 Katy works as a clerical assistant for a small company. Each morning, she collects the company's post from a secure box in the nearby Royal Mail sorting office.

Katy's supervisor asks her to keep a daily record of the number of letters that she collects.

Her records for a period of 175 days are summarised in the table.

| Daily number of letters <br> $(\boldsymbol{x})$ | Number of days <br> $(\boldsymbol{f})$ |
| :---: | :---: |
| $0-9$ | 5 |
| $10-19$ | 16 |
| 20 | 23 |
| 21 | 27 |
| 22 | 31 |
| 23 | 34 |
| 24 | 16 |
| $25-29$ | 10 |
| $30-34$ | 5 |
| $35-39$ | 3 |
| $40-49$ | 4 |
| 50 or more | 1 |
| Total | 175 |

(a) For these data:
(i) state the modal value;
(ii) determine values for the median and the interquartile range.
(b) The most letters that Katy collected on any of the 175 days was 54. Calculate estimates of the mean and the standard deviation of the daily number of letters collected by Katy.
(c) During the same period, a total of 280 letters was also delivered to the company by private courier firms.

Calculate an estimate of the mean daily number of all letters received by the company during the 175 days.

3 The table shows the maximum weight, $y_{A}$ grams, of Salt $A$ that will dissolve in 100 grams of water at various temperatures, $x^{\circ} \mathrm{C}$.

| $\boldsymbol{x}$ | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}_{\boldsymbol{A}}$ | 20 | 35 | 48 | 57 | 77 | 92 | 101 | 111 | 121 | 137 | 159 | 182 |

(a) Calculate the equation of the least squares regression line of $y_{A}$ on $x$. (4 marks)
(b) The data in the above table are plotted on the scatter diagram on page 4.

Draw your regression line on this scatter diagram.
(c) For water temperatures in the range $10^{\circ} \mathrm{C}$ to $80^{\circ} \mathrm{C}$, the maximum weight, $y_{B}$ grams, of Salt $B$ that will dissolve in 100 grams of water is given by the equation

$$
y_{B}=60.1+0.255 x
$$

(i) Draw this line on the scatter diagram.
(ii) Estimate the water temperature at which the maximum weight of Salt $A$ that will dissolve in 100 grams of water is the same as that of Salt B.
(1 mark)
(iii) For Salt $A$ and Salt B, compare the effects of water temperature on the maximum weight that will dissolve in 100 grams of water. Your answer should identify two distinct differences.

Temperatures and Maximum Weights


4 A survey of the 640 properties on an estate was undertaken. Part of the information collected related to the number of bedrooms and the number of toilets in each property.

This information is shown in the table.

|  | Number of toilets |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ or <br> more | Total |
|  | $\mathbf{1}$ | 46 | 14 | 0 | 0 | 60 |
| Number of <br> bedrooms | $\mathbf{3}$ | 24 | 67 | 23 | 0 | 114 |
|  | $\mathbf{3}$ | 7 | 72 | 99 | 16 | 194 |
|  | $\mathbf{4}$ | 0 | 19 | 123 | 48 | 190 |
|  | $\mathbf{5}$ or <br> more | 0 | 0 | 11 | 71 | 82 |

(a) A property on the estate is selected at random.

Find, giving your answer to three decimal places, the probability that the property has:
(i) exactly 3 bedrooms;
(ii) at least 2 toilets;
(iii) exactly 3 bedrooms and at least 2 toilets;
(iv) at most 3 bedrooms, given that it has exactly 2 toilets.
(b) Use relevant answers from part (a) to show that the number of toilets is not independent of the number of bedrooms.
(c) Three properties are selected at random from those on the estate which have exactly 3 bedrooms.

Calculate the probability that one property has 2 toilets, one has 3 toilets and the other has at least 4 toilets. Give your answer to three decimal places.

5 A general store sells lawn fertiliser in 2.5 kg bags, 5 kg bags and 10 kg bags.
(a) The actual weight, $W$ kilograms, of fertiliser in a 2.5 kg bag may be modelled by a normal random variable with mean 2.75 and standard deviation 0.15 .

Determine the probability that the weight of fertiliser in a 2.5 kg bag is:
(i) less than 2.8 kg ;
(ii) more than 2.5 kg .
(b) The actual weight, $X$ kilograms, of fertiliser in a 5 kg bag may be modelled by a normal random variable with mean 5.25 and standard deviation 0.20 .
(i) Show that $\mathrm{P}(5.1<X<5.3)=0.372$, correct to three decimal places. (2 marks)
(ii) A random sample of four 5 kg bags is selected. Calculate the probability that none of the four bags contains between 5.1 kg and 5.3 kg of fertiliser.
(c) The actual weight, $Y$ kilograms, of fertiliser in a 10 kg bag may be modelled by a normal random variable with mean 10.75 and standard deviation 0.50 .

A random sample of six 10 kg bags is selected. Calculate the probability that the mean weight of fertiliser in the six bags is less than 10.5 kg .
(4 marks)

6 A bin contains a very large number of paper clips of different colours. The proportion of each colour is shown in the table.

| Colour | White | Yellow | Green | Blue | Red | Purple |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proportion | 0.15 | 0.15 | 0.20 | 0.15 | 0.22 | 0.13 |

(a) Packets are filled from the bin. Each filled packet contains exactly 30 paper clips which may be considered to be a random sample.

Use binomial distributions to determine the probability that a filled packet contains:
(i) exactly 2 purple paper clips;
(ii) a total of more than 10 red or purple paper clips;
(iii) at least 5 but at most 10 green paper clips.
(b) Jumbo packets are also filled from the bin. Each filled jumbo packet contains exactly 100 paper clips.
(i) Assuming that the number of paper clips in a jumbo packet may be considered to be a random sample, calculate the mean and the variance of the number of red paper clips in a filled jumbo packet.
(2 marks)
(ii) It is claimed that the proportion of red paper clips in the bin is greater than 0.22 and that jumbo packets do not contain random samples of paper clips.

An analysis of the number of red paper clips in each of a random sample of 50 filled jumbo packets resulted in a mean of 22.1 and a standard deviation of 4.17.

Comment, with numerical justification, on each of the two claims.
(3 marks)

7 The volume of bleach in a 5-litre bottle may be modelled by a random variable with a standard deviation of 75 millilitres.

The volume, in litres, of bleach in each of a random sample of 36 such bottles was measured. The 36 measurements resulted in a total volume of 181.80 litres and exactly 8 bottles contained less than 5 litres.
(a) Construct a $98 \%$ confidence interval for the mean volume of bleach in a 5 -litre bottle.
(5 marks)
(b) It is claimed that the mean volume of bleach in a 5 -litre bottle exceeds 5 litres and also that fewer than 10 per cent of such bottles contain less than 5 litres.

Comment, with numerical justification, on each of these two claims.
(c) State, with justification, whether you made use of the Central Limit Theorem in constructing the confidence interval in part (a).
(1 mark)

